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Forced convection heat transfer of Couette–Poiseuille flow of nonlinear viscoelastic fluids between parallel plates

S.H. Hashemabadi [a], S.Gh. Etemad [b,*], J. Thibault [c]

[a] Department of Chemical Engineering, Iran University of Science and Technology, Tehran, Iran
[b] Department of Chemical Engineering, Isfahan University of Technology, Isfahan 84155, Iran
[c] Department of Chemical Engineering, University of Ottawa, Canada

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Abstract

Heat transfer to viscoelastic fluids is frequently encountered in various industrial processing. In this investigation an analytical solution was obtained to predict the fully developed, steady and laminar heat transfer of viscoelastic fluids between parallel plates. One of the plates was stationary and was subjected to a constant heat flux. The other plate moved with constant velocity and was insulated. The simplified Phan-Thien–Tanner (SPTT) model, believed to be a more realistic model for viscoelastic fluids, was used to represent the rheological behavior of the fluid. The energy equation was solved for a wide range of Brinkman number, dimensionless viscoelastic group, and dimensionless pressure drop. Results highlight the strong effects of these parameters on the heat transfer rate.

Keywords: Viscoelastic fluid; Parallel plates; Couette; Poiseuille; Single screw extruder; Phan-Thien–Tanner model

1. Introduction

The flow and heat transfer behavior of viscoelastic fluids between parallel plates is of special engineering interest. One of the most important processes in industry is extrusion. Since the gap between the barrel and the screw of extruder is small, assuming a fluid flowing between parallel plates leads to representative results. There exist a large number of parameters in the extrusion processes which influence significantly the production rate and the quality of the final product. The rheological behavior of the processing fluid plays a key role in the flow and heat transfer through the extruders. The fluids employed in extruders exhibit viscoelastic non-Newtonian behavior. One of the frequently used viscoelastic model to represent the rheological behavior of these fluids is the Phan-Thien–Tanner (PTT) model which was derived by network theory (Phan-Thien and Tanner [1], Phan-Thien [2]). Another parameter which bears great significance on heat transfer is viscous dissipation. When the viscosity of the fluid and/or the velocity gradient is high, the dissipation term becomes important and its order of magnitude is comparable with the convection and diffusion terms of the energy equation.

Several investigations have been done in the field of non-Newtonian fluid flow and heat transfer between parallel plates. Etemad et al. [3] solved the simultaneously developing case of the motion and energy equation for power law fluids between parallel stationary plates when the variation of viscosity with temperature and viscous dissipation could not be neglected. They solved the problem numerically using finite element method and, as a special case, calculated the flow and heat transfer characteristics for fully developed conditions. Chou et al. [4] considered the channel non-Newtonian heat transfer including viscous dissipation

* Corresponding author. Tel.: +98-311-3915625; fax: +98-311-3912677.
E-mail address: etemad@cc.iut.ac.ir (S.Gh. Etemad).

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Nomenclature

Br	Brinkman number
C_p	heat capacity
De	Deborah number $\left(\frac{\lambda V}{H}\right)$
G	dimensionless pressure gradient
H	gap between of the parallel plate
k	thermal conductivity
Nu	Nusselt number $\frac{h(2H)}{k}$
p	pressure
q	heat flux
Re	Reynolds number $\left(\frac{\rho\bar{u}(2H)}{\eta}\right)$
T	temperature
u	velocity
V	velocity of moving plate
x	axial coordinate
y	lateral coordinate
Greek symbols	
α	defined in Appendix A
β	defined in Appendix A
δ	defined in Appendix A
ε	extensional parameter of the PTT model
η	viscosity coefficient of the PTT model
θ	dimensionless temperature
λ	relaxation time in the PTT model
ρ	density
τ	stress tensor
Subscripts	
m	mean value
N	Newtonian fluid
0	value at the stationary plate
w	value at the heated plate
Superscripts	
$*$	refers to dimensionless quantities
–	refers to the average value

for stationary parallel plates. Patel and Ingham [5] obtained an analytical solution for the mixed convection of power law model non-Newtonian fluid between parallel plates with a constant wall temperature boundary condition. Olek [6] used a specific eigenfunction expansion to derive an exact solution for heat transfer between two stationary parallel plates. Lawal and Dilhan [7] developed an analytical solution for the viscoplastic (Hershel Bulkley) model fluid flow and heat transfer in nonisothermal screw extrusion processing. Davaa et al. [8] solved numerically the fluid flow and heat transfer equation for a modified power law fluid in Couette–Poiseuille laminar flow between parallel plates. Recently Pinho et al. [9] considered the problems of laminar axisymmetric forced convection of simplified PTT viscoelastic model fluid flowing between parallel plates with an imposed constant wall heat flux.

So far, heat transfer between parallel plates has not yet been considered for the case of a PTT rheological model fluid flowing between parallel plates with one plate moving at constant speed. The aim of the present study is to develop an analytical solution for Couette–Poiseuille laminar flow heat transfer of a PTT viscoelastic model fluid flowing between parallel plates and for which viscous dissipation cannot be neglected.

2. Mathematical formulation

Fig. 1 presents a schematic diagram of the fluid flow and heat transfer domains. The channel consists of two parallel infinite plates. One plate is stationary and the other is moving with constant velocity. The problem under consideration is steady, laminar, and hydrodynamically and thermally fully developed. For internal laminar flow the entrance length depends on the Reynolds number and polymeric liquids usually have high viscosity so the Reynolds number for this process is very small, therefore entrance length can be neglected and fully developed flow can be assumed. The physical fluid properties are assumed constant. The governing energy equation describing this problem, with the assumption of appreciable viscous dissipation and negligible axial heat conduction, can be represented by the following equation:

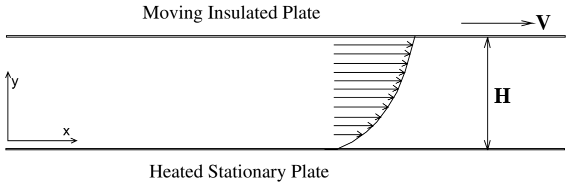

Fig. 1. Schematic diagram of flow domain.

$$\rho c_p u \frac{\partial T}{\partial x} = k \frac{\partial^2 T}{\partial y^2} + \tau \frac{\mathrm{d}u}{\mathrm{d}y} \tag{1}$$

In this study, the stationary plate is subjected to a constant heat flux, and the moving plate is insulated. The boundary conditions are

$$y = 0, \quad -k \frac{\partial T}{\partial y} = q_{\mathrm{w}} \tag{2}$$

$$y = H, \quad \frac{\partial T}{\partial y} = 0 \tag{3}$$

When the temperature profile is fully developed one can write (Bejan [10]):

$$\frac{\partial}{\partial x}\left(\frac{T_w - T}{T_w - T_m}\right) = 0 \tag{4}$$

Therefore

$$\frac{\partial T}{\partial x} = \frac{\partial T_m}{\partial x} = \text{Const.} \tag{5}$$

where T_m is the bulk temperature of the fluid. Performing an energy balance over an infinitesimal element dx of fluid, the following expression is obtained (Shah and London [11]):

$$\frac{\partial T_m}{\partial x} = \frac{1}{\rho c_p \bar{u} H}\left(q_w + \int_0^H \tau \frac{du}{dy} dy\right) \tag{6}$$

Combining Eqs. (1), (5) and (6), the dimensionless energy equation is derived as follows:

$$\frac{\partial^2 \theta}{\partial y^{*2}} + Br\left(\tau^* \frac{du^*}{dy^*} - \frac{u^*}{\bar{u}^*}\int_0^1 \tau^* \frac{du^*}{dy^*} dy^*\right) = \frac{u^*}{\bar{u}^*} \tag{7}$$

where

$$y^* = \frac{y}{H}, \quad u^* = \frac{u}{V}, \quad \tau^* = \frac{\tau H}{\eta V} \tag{8}$$

For constant heat flux boundary condition:

$$\theta = \frac{T - T_m}{q_w H/k}, \quad Br = \frac{\eta V^2}{H q_w} \tag{9}$$

The Nusselt number at the heated (stationary) plate is

$$Nu = \frac{2}{\theta_w} \tag{10}$$

The dimensionless boundary conditions become

$$y^* = 0, \quad \frac{\partial \theta}{\partial y^*} = -1 \tag{11}$$

$$y^* = 1, \quad \frac{\partial \theta}{\partial y^*} = 0 \tag{12}$$

3. Analytical solution

In this investigation the simplified Phan-Thien–Tanner (SPTT) constitutive equation with linearized stress coefficient was employed. This rheological equation (SPTT) can be described by the following expression (Bird [12]):

$$Z(\mathrm{tr}\,\tau)\tau + \lambda\tau_{(1)} = -\eta\dot{\gamma} \tag{13}$$

where η is the viscosity coefficient of the model, λ is the relaxation time and $\mathrm{tr}\,\tau$ is the trace of the stress tensor τ. $\tau_{(1)}$ is the convected time derivative of the stress tensor. The stress coefficient Z has an exponential form that may be linearized when the deformation rate of fluid's elements is small:

$$Z(\mathrm{tr}\,\tau) = 1 - \varepsilon\lambda\frac{\mathrm{tr}\,\tau}{\eta} \tag{14}$$

Performing the stress tensor component analysis and solving of the continuity and momentum equations (Hashemabadi et al. [13]) the dimensionless profiles of velocity (u^*), shear rate $\left(\dot{\gamma}^*_{yx}\right)$, and shear stress $\left(\tau^*_{yx}\right)$ were obtained:

$$u^* = \frac{1}{2}y^*(Gy^* - 2\tau_0^*) + \underline{\frac{1}{4}\varepsilon De^2 y^*(Gy^* - 2\tau_0^*)\left[(Gy^*)^2 + (Gy^* - 2\tau_0^*)^2\right]} \tag{15}$$

$$\dot{\gamma}^*_{yx} = \frac{du^*}{dy^*} = \underline{\left(1 + 2\varepsilon De^2(Gy^* - \tau_0^*)^2\right)}(Gy^* - \tau_0^*) \tag{16}$$

$$\tau^*_{yx} = \tau_0^* - Gy^* \tag{17}$$

where

$$De = \frac{\lambda V}{H}, \quad G = \frac{H^2}{\eta v}\left(\frac{dp}{dx}\right) \tag{18}$$

G, De and τ_0^* are the dimensionless pressure group, Deborah number and shear stress on the stationary plate, respectively. The underlined terms arise from the SPTT model. For Newtonian fluids, the underlined term in Eq. (15) is equal to zero whereas it is equal to one in Eq. (16). Solving the energy equation (7) for constant heat flux boundary condition, using the dimensionless profiles of Eqs. (15)–(17) leads to

$$\theta - \theta_w = \sum_{i=1}^{6}(a_i + b_i Br)y^{*i} \tag{19}$$

The coefficients a_i and b_i are function of the dimensionless pressure gradient (G) and the viscoelastic group (εDe^2). Coefficients a_i and b_i are given in Table 1 of Appendix A.

The dimensionless wall temperature θ_w, using the definition of the mean temperature, is given by following equation (Shah and London [11]):

$$\theta_w = \frac{1}{\bar{u}^*}\int_0^1 (\theta_w - \theta)u^* dy^* \tag{20}$$

By substitution of Eqs. (15) and (19) into Eq. (20), the dimensionless wall temperature is calculated:

$$\theta_w = c + dBr \tag{21}$$

where

$$c = \frac{1}{\bar{u}^{*2}}\sum_{j=1}^{3}\left(\sum_{i=0}^{2j} c_{ij} G^{(2j-i)}\tau_{0l}^{*i}\right)(\varepsilon De^2)^{(j-1)} \tag{22}$$

$$d=\frac{1}{\bar{u}^*}\sum_{j=1}^{3}\left(\sum_{i=0}^{2j+1}d_{1ij}G^{(2j-i+1)}\tau_{0l}^{*i}\right)(\varepsilon De^2)^{(j-1)}+\frac{1}{\bar{u}^{*2}}\sum_{j=2}^{5}\left(\sum_{i=0}^{2j}d_{2ij}G^{(2j-i)}\tau_{0l}^{*i}\right)(\varepsilon De^2)^{(j-2)} \tag{23}$$

$$Nu_N=\frac{(G-6)^2}{\frac{13}{70}G^2-\frac{13}{5}G+\frac{48}{5}-Br\left(\frac{3}{1120}G^4-\frac{3}{35}G^3+\frac{117}{140}G^2-\frac{12}{5}G+\frac{21}{10}\right)} \tag{25}$$

The constant coefficients c_{ij}, d_{1ij} and d_{2ij} are given in Appendix A (Table 2). Combining Eqs. (10) and (21), the Nusselt number at the heated plate is obtained:

$$Nu=\frac{2}{c+dBr} \tag{24}$$

For Newtonian fluids, the particular equation of the Nusselt number is

For Newtonian fluids when both plates are stationary, G approaches infinity, and the value of the Nusselt number is equal to 5.385.

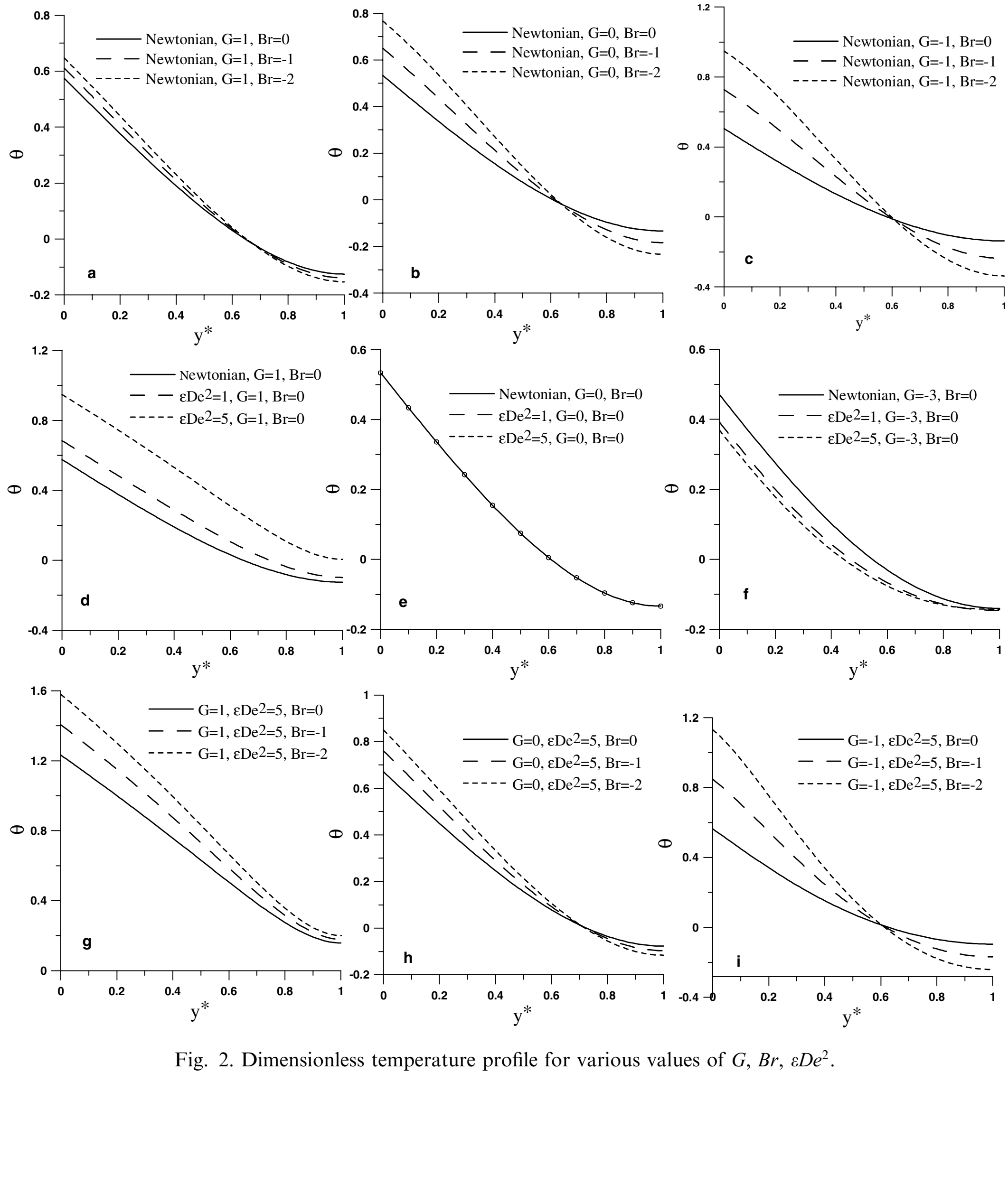

Fig. 2. Dimensionless temperature profile for various values of G, Br, εDe^2.

4. Discussion

4.1. Effect of G and εDe^2 on the Nusselt number

Figs. 2(d)–(f) and 3a present the variation of the dimensionless temperature profile and the Nusselt number with the dimensionless viscoelastic group εDe^2 respectively, for different values of G when the viscous dissipation is negligible. Based on Eq. (10), the Nusselt number is only related to the dimensionless wall temperature. Negative and positive values of G correspond to the combination of drag and pressure flow (Couette–Poiseuille) whereas zero corresponds to pure drag (Couette) flow. For positive values of G, backflow exists in the channel. From Fig. 3(a), when εDe^2 is zero, the flow behaves as a Newtonian fluid and the effect of G on the Nusselt number is negligible. For positive values of G, increasing the dimensionless viscoelastic group increases the dimensionless wall temperature (Fig. 2(d)), and a decrease in the Nusselt number is observed. For negative values of G, this behavior is reversed (Fig. 2(f)). For $G = 0$, the dimensionless wall temperature is independent of the viscoelastic group (Fig. 2(e)) and a constant Nusselt number prevails.

4.2. Effect of viscous dissipation on the Nusselt number

The effect of viscous dissipation is very important when the viscosity is high or for high shear flows. The Brinkman number is commonly used as a parameter to characterize the relative importance of viscous dissipation compared to the external heat transfer.

The effect of viscous dissipation on the temperature profile for different values of G and εDe^2 are presented in Fig. 2. Fig. 3 also shows the effect of the Brinkman number on the Nusselt number for different values of G and εDe^2. Because the shear rate is highest near the fixed plate, the effect of viscous heating is most significant in that region. The dimensionless wall temperature increases with an increase in the viscous dissipation and,

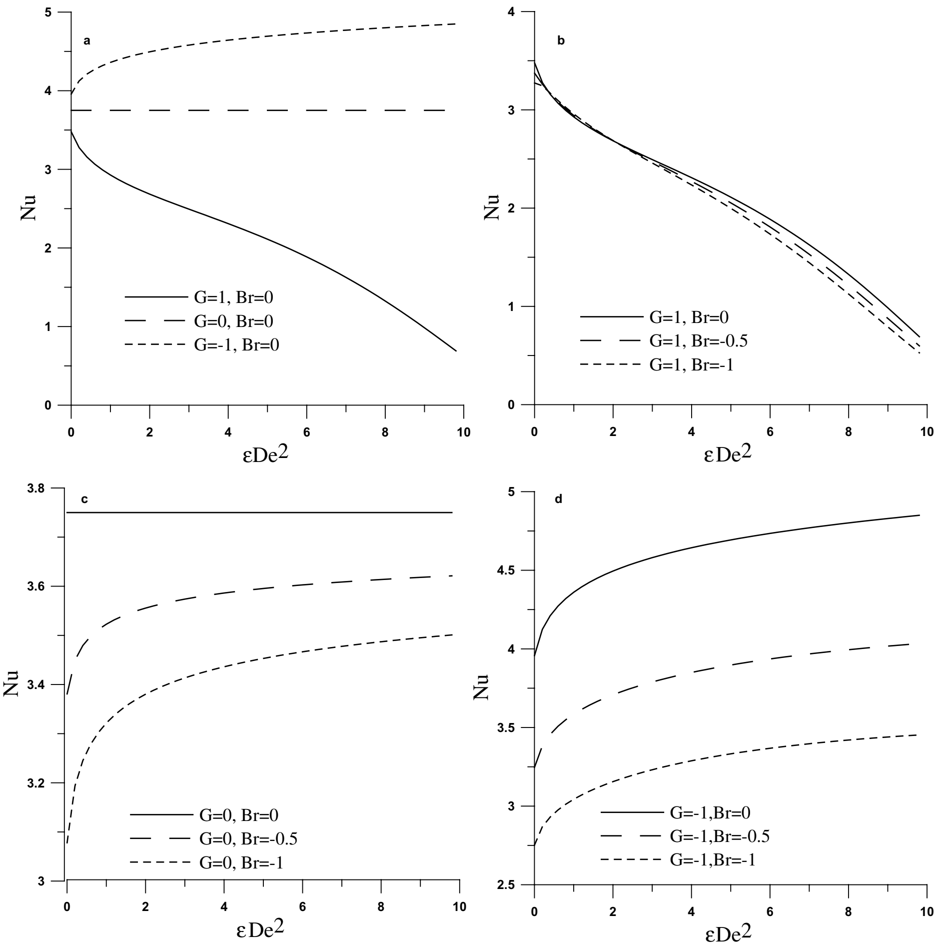

Fig. 3. The variation of the Nusselt number with εDe^2.

as a result, the Nusselt number decreases. The magnitude of the reduction depends on the values of G. When G is positive, the effect of viscous dissipation on the Nusselt number is not as significant as for null and negative values of G, as observed by the influence of the Brinkman number on the Nusselt number.

5. Conclusion

In this investigation, an analytical solution was derived for heat transfer between parallel plates under steady, laminar, and hydrodynamically and thermally developed flow. The lower plate was at rest and subjected to a constant heat flux whereas the upper plate was moving at constant speed and insulated. The simplified Phan-Thien–Tanner (SPTT) model was used as the viscoelastic model. This investigation included the effect of viscous dissipation on heat transfer. Results emphasize the significant effect of viscous heating on the Nusselt number. Increasing the Brinkman number decreases the heat transfer to the fluid. Also the product of the elongational parameter and the Deborah number is an important parameter and, depending on the values of G, may greatly influence the heat transfer coefficient.

Appendix A

Table 1. Coefficients of polynomial equation (19)

$a_1 = -1$	$b_1 = 0$
$a_2 = 0$	$b_2 = \frac{1}{2}\delta\tau_0^{*2}$
$a_3 = -\frac{1}{6\bar{u}^*}\delta\tau_0^*$	$b_3 = a_3(\beta + 2\bar{u}^* G) - \frac{2}{3}\varepsilon De^2 \tau_{0l}^{*3} G$
$a_4 = \frac{G}{24\bar{u}^*}\left(\delta + 4\tau_0^{*2}\varepsilon De^2\right)$	$b_4 = \frac{G^2}{12}\left(\delta + 10\tau_0^{*2}\varepsilon De^2\right) + \beta a_4$
$a_5 = -\frac{\varepsilon De^2 \tau_0^* G^2}{10\bar{u}^*}$	$b_5 = a_5(4\bar{u}^* G + \beta)$
$a_6 = -\frac{\varepsilon De^2 G^3}{60\bar{u}^*}$	$b_6 = a_6(4\bar{u}^* + \beta)$

where

$$\tau_0^* = \frac{\alpha}{6\varepsilon De^2} - \frac{1}{2\alpha}(\varepsilon De^2 G^2 + 2) + \frac{1}{2}G,$$

$$\bar{u}^* = -\frac{1}{2}\tau_0^* + \frac{1}{6}G + \varepsilon De^2\left(\frac{1}{10}G^3 - \frac{1}{2}\tau_0^* G^2 + \tau_0^{*2} G - \tau_0^{*3}\right),$$

$$\alpha = \left[\left(-54 + 3\left[3\frac{(\varepsilon De^2)^3 G^6 + 6(\varepsilon De^2)^2 G^4 + (12G^2 + 108)(\varepsilon De^2) + 8}{\varepsilon De^2}\right]^{\frac{1}{2}}\right)(\varepsilon De^2)^2\right]^{\frac{1}{3}},$$

$$\beta = \left(-\frac{2}{5}G^4 + 2\tau_0^* G^3 - 4\tau_0^{*2} G^2 + 4\tau_0^{*3} G - 2\tau_0^{*4}\right)\varepsilon De^2 - \frac{1}{3}G^2 + \tau_0^* G - \tau_0^{*2}.$$

Table 2. Coefficients of polynomial equation (22)

	ji	0	1	2	3	4	5	6	7	8	9	10
c	1	$\frac{1}{56}$	$-\frac{7}{72}$	$\frac{2}{15}$								
	2	$\frac{5}{216}$	$-\frac{7}{40}$	$\frac{109}{210}$	$-\frac{7}{9}$	$\frac{8}{15}$						
	3	$\frac{1}{132}$	$-\frac{11}{150}$	$\frac{19}{60}$	$-\frac{4}{5}$	$\frac{263}{216}$	$-\frac{7}{6}$	$\frac{8}{15}$				
d_1	1	$\frac{1}{28}$	$-\frac{7}{36}$	$\frac{41}{120}$	$-\frac{5}{24}$							
	2	$\frac{5}{72}$	$-\frac{21}{40}$	$\frac{337}{210}$	$-\frac{77}{30}$	$\frac{11}{5}$	$-\frac{5}{6}$					
	3	$\frac{1}{33}$	$-\frac{22}{75}$	$\frac{19}{15}$	$-\frac{16}{5}$	$\frac{536}{105}$	$-\frac{77}{15}$	$\frac{91}{30}$	$-\frac{5}{6}$			

Table 2 (*continued*)

	ji	0	1	2	3	4	5	6	7	8	9	10
d_2	2	$-\frac{1}{168}$	$\frac{19}{378}$	$-\frac{67}{420}$	$\frac{83}{360}$	$-\frac{2}{15}$						
	3	$-\frac{337}{22680}$	$\frac{59}{378}$	$-\frac{466}{675}$	$\frac{12703}{7560}$	$-\frac{613}{252}$	$\frac{367}{180}$	$-\frac{4}{5}$				
	4	$-\frac{7}{594}$	$\frac{881}{5940}$	$-\frac{34789}{41580}$	$\frac{52891}{18900}$	$-\frac{28943}{4725}$	$\frac{3799}{420}$	$-\frac{2797}{315}$	$\frac{97}{18}$	$-\frac{8}{5}$		
	5	$-\frac{1}{330}$	$\frac{367}{8250}$	$-\frac{167}{550}$	$\frac{2107}{1650}$	$-\frac{14153}{3850}$	$\frac{3982}{525}$	$-\frac{11959}{1050}$	$\frac{432}{35}$	$-\frac{977}{105}$	$\frac{67}{15}$	$-\frac{16}{15}$

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